Fall 2011, Math 290, Midterm I Practice

Name (Print) and Student ID:

Signature:

There are a total of 100 points on this 50 minutes exam. This contains 7 pages (including this cover page) and 9 problems. Check to see if any page is missing. Enter all requested information on the top of this page. Calculators may be needed. Please turn off cell phones. You are allowed to bring one-half of one single-sided 8.5×11 inch page of notes, in your own handwriting, to the exam. Do not give numerical approximations to quantities such as $\sin 5, \pi, e$ or $\sqrt{2}$. However you should simply $\sin \frac{\pi}{2} = 1$ and $e^0 = 1$, etc.

The following rules apply:

- To get full credit for a problem you must show the details of your work, in a reasonably neat and coherent way, in the space provided. Answers unsupported by an argument will get little credit. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically CORRECT and carefully and legibly written.
- NO books. No computers. Do all of your calculations on this test paper.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Problem 1. (10 points) Determine which of the following equations is a linear equation?

(A) $xy + z = 0$
(B) $\dfrac{1}{x} + y + z = 1$
(C) $\sin x + y + z = 2$
(D) $\pi x + y - 1 = 0$
(E) $x + y^2 + z = 3$.

Problem 2. (10 points) If $x = s$, $y = t$ and $z = 5 - 2s - t$, where s and t are parameters, then the corresponding linear equation of x, y and z is

(A) $2x + y + z = 5$
(B) $x + y + z = 0$
(C) $x + y = s + t$
(D) $xy + z + 2x + y = st + 5$
(E) None of the above is correct.

Problem 3. (10 points) Suppose $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 4-t^2 & 3 \end{pmatrix}$ is the augmented matrix for a linear system of three linear equations in x, y and z. Which statement is CORRECT?

(A) If $t = 2$ or $t = -2$, the system has no solutions.

(B) For any t, this system is consistent.

(C) If $t = 1$, this system has exactly one solution.

(D) If $t = -1$, this system has exactly one solution.

(E) None of the above is correct.

Problem 4. (10 points) Given three points $(1, 3)$, $(0, 1)$ and $(-1, 1)$. The graph of $p(x) = a_0 + a_1x + a_2x^2$ passes through all these three points. Which statement below is CORRECT?

(A) $a_0 = 2, a_1 = 1$ and $a_2 = 0$.

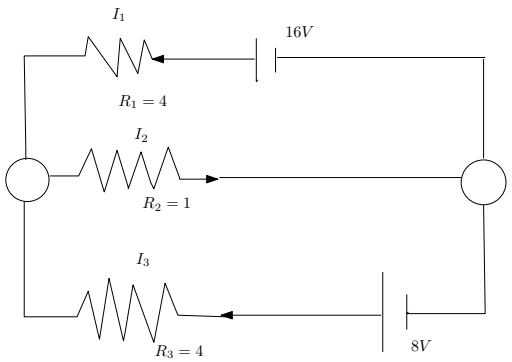
(B) $a_0 = 3, a_1 = 0$ and $a_2 = 0$.

(C) $a_0 = 1, a_1 = 1$ and $a_2 = 1$.

(D) There are infinitely many triples (a_0, a_1, a_2) which satisfy the condition.

(E) None of the above is correct.

Problem 5. (10 points) For the electrical network shown below, which of the following CORRECTLY represents the linear system of currents I_1, I_2 and I_3?



(A) $\begin{cases} I_1 + I_2 + I_3 = 0 \\ 4I_1 + I_2 = 16 \\ I_2 + 4I_3 = 8. \end{cases}$

(B) $\begin{cases} I_1 - I_2 + I_3 = 0 \\ 4I_1 + I_2 = 16 \\ I_2 + 4I_3 = 8. \end{cases}$

(C) $4I_1 + I_2 = 16.$

(D) $\begin{cases} 4I_1 + I_2 = 16 \\ I_2 + 4I_3 = 8. \end{cases}$

(E) None of the above is correct.

Problem 6. (10 points) For the CORRECT linear system in **Problem 5**, which of the following statements is CORRECT?

(A) The system is inconsistent.

(B) The system has infinitely many solutions.

(C) The system can be represented by matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix} \times \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 16 \\ 8 \end{pmatrix}.$$

(D) The augmented matrix for the system is

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 4 & 1 & 0 & 16 \\ 0 & 1 & 4 & 8 \end{pmatrix}.$$

(E) None of the above is correct.

Problem 7. (10 points) Consider the matrices

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}.$$

Which of the following statements is WRONG?

(A) $AC = BC$.

(B) Because $AC = BC$, $A = B$.

(C) $A + B + C = \begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix}$.

(D) $A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

(E) $A^3 = A$.

Problem 8. (10 points) Let $A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 4 & 1 \\ -3 & 1 & 1 \end{pmatrix}$. Which of the following statements is WRONG?

(A) $A^T = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 4 & 1 \\ -3 & 1 & 1 \end{pmatrix}$

(B) Both AA^T and A^TA are symmetric.

(C) A is symmetric.

(D) $\left((A^T)^T\right)^T = A$.

(E) None of the above is correct.

Problem 9. Consider the linear system

$$\begin{cases} x_1 & & -3x_3 & = 2 \\ 3x_1 & +x_2 & -2x_3 & = 5 \\ 2x_1 & +2x_2 & +x_3 & = 4 \end{cases}$$

(a). (5 points). Write this system represented by matrix $Ax = b$ and then write the down the augmented matrix.

(b). (10 points). Following the augmented matrix in (a), use the method of Gaussian elimination to reduce it to the row-echelon form.

(c). (5 points). Following the row-echelon form in (b), reduces it further to the reduced row-echelon form and then write down the solution set for this linear system.