Fall 2011, Math 290, Midterm II Practice Key

Name (Print): (first)____________ (last)____________

Signature:

There are a total of 100 points on this 50 minutes exam. This contains 6 pages (including this cover page) and 8 problems. Check to see if any page is missing. Enter all requested information on the top of this page. Calculators may be needed. Please turn off cell phones. You are allowed to bring one-half of one single-sided 8.5×11 inch page of notes, in your own handwriting, to the exam. Do not give numerical approximations to quantities such as $\sin 5, \pi, e$ or $\sqrt{2}$. However you should simply $\sin \frac{\pi}{2} = 1$ and $e^0 = 1$, etc.

The following rules apply:

- To get full credit for a problem you must show the details of your work, in a reasonably neat and coherent way, in the space provided. Answers unsupported by an argument will get little credit. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically CORRECT and carefully and legibly written.
- NO books. No computers. Do all of your calculations on this test paper.

Problem	Score
1	
2	
3	
Total	

Write down the answers to the problems of multiple choice here.

Problem	1	2	3	4	5	6
Your answer	D	E	B	B	C	B

Problem 1. (10 points) Determine which of the following statements is wrong?

(A) The inverse of a nonsingular matrix is unique.

(B) If the matrix A, B and C satisfy $BA = CA$, and A is invertible, then $B = C$.

(C) If A can be row-reduced to the identity matrix, then A is nonsingular.

(D) The matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible if $ab - cd \neq 0$.

(E) If A is invertible, then the system of linear equations $Ax = b$ has a unique solution.

Problem 2. (10 points) Determine which of the following is right?

(A) The determinant of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ab - cd$.

(B) If $A = (-5)$, then $|A| = 5$.

(C) The ij-cofactor of a square matrix A is the matrix defined by deleting the $i-$th row and the $j-$th column of A.

(D) To find the determinant of a triangular matrix, add the entries on the diagonal line of this matrix.

(E) When expanding by cofactors in evaluating the determinant of a matrix, one does not need to evaluate the cofactors of zero entries.

Problem 3. (10 points) Let A be a square matrix. Determine which statement is wrong?

(A) Interchanging two rows of A changes the signs of $|A|$.

(B) Multiplying a row of A by -5 results in $|A|$ being multiplied by 5.

(C) If one row of A is a multiple of another row, then $|A| = 0$.

(D) A is column-equivalent to B if B can be obtained from A by performing elementary column operations on A.

(E) None of the above.

Problem 4. (10 points) Determine which of the following statements is wrong?

(A) If A is invertible, then $A^{-1} = \dfrac{\mathrm{adj}(A)}{|A|}$, where $\mathrm{adj}(A)$ is the adjoint of A.

(B) For any matrix A, the adjoint of A always exists.

(C) If A is a $n \times n$ matrix, then $\mathrm{adj}(A)$ is also a $n \times n$ matrix.

(D) If A is a $n \times n$ matrix, then $|A \cdot \mathrm{adj}(A)| = |A|^n$.

(E) None of the above.

Problem 5. (10 points) Let $u = (0, 1, 4), v = (-1, 1, 2)$ and $w = (3, 1, 2)$ be vectors in $\mathbb{R}^3$. Determine which of the following statements is wrong?

(A) $x = (-1, -2, 2)$ can be expressed as a linear combination of u, v and w.

(B) The zero vector can be expressed as a linear combination of u, v and w.

(C) The vector $(1, 1, 1)$ can not be expressed as a linear combination of u, v and w.

(D) Any vector in $\mathbb{R}^3$ can be expressed as a linear combination of u, v and w.

(E) None of the above.

Problem 6. (10 points) Determine which of the following statements is wrong?

(A) The set of all 2×2 symmetric matrices forms a subspace of the vector space $M_{2,2}$ with the standard operation of matrix addition and scalar multiplication.

(B) The set of all singular matrices of order 2 forms a subspace of the vector space $M_{2,2}$ with the standard operation of the matrix addition and scalar multiplication.

(C) The set $W = \{(x_1, x_2) : x_1 \geq 0\}$ with the usual operations is not a subspace of $\mathbb{R}^2$.

(D) The set $W = \{(x_1, x_2) : x_1^2 + x_2^2 = 1\}$ with the usual operations is not a subspace of $\mathbb{R}^2$.

(E) None of the above.

Problem 7. Given the system of linear equations

$$\begin{cases} x_1 + x_2 - x_3 & = 4 \\ 2x_1 - x_2 + x_3 & = 6 \\ 3x_1 - 2x_2 - 2x_3 & = 0. \end{cases}$$

(a). (5 points) Write the system in the form $Ax = b$. Evaluate $|A|$.

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 3 & -2 & -2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}.$$

Let A be the coefficient matrix. Then by applying row operations,

$$|A| = \begin{matrix} 1 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & -5 & 1 \end{matrix} = \begin{matrix} -3 & 3 \\ -5 & 1 \end{matrix} = -3 + 15 = 12.$$

(b). (5 points) Find A^{-1} by using the elementary row operations.

$$(A|I) = \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 3 & -2 & -2 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -3 & 3 & -2 & 1 & 0 \\ 0 & -5 & 1 & -3 & 0 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2/3 & -1/3 & 0 \\ 0 & -5 & 1 & -3 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2/3 & -1/3 & 0 \\ 0 & 0 & -4 & 1/3 & -5/3 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & -1/12 & 5/12 & -1/4 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & 11/12 & 5/12 & -1/4 \\ 0 & 1 & 0 & 7/12 & 1/12 & -1/4 \\ 0 & 0 & 1 & -1/12 & 5/12 & -1/4 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 1 & 0 & 7/12 & 1/12 & -1/4 \\ 0 & 0 & 1 & -1/12 & 5/12 & -1/4 \end{pmatrix}.$$

So

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ 7 & 1 & -3 \\ -1 & 5 & -3 \end{pmatrix}.$$

(c). (5 points) Find A^{-1} by using the formula $A^{-1} = \frac{\mathrm{adj}(A)}{|A|}$.
One first computes

$$c_{11} = 4, c_{12} = 7, c_{13} = -1, c_{21} = 4, c_{22} = 1, c_{23} = 5$$

and

$$c_{31} = 0, c_{32} = -3, c_{33} = -3.$$

Then

$$A^{-1} = \frac{1}{|A|}C = \frac{1}{12}\begin{pmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{pmatrix} = \frac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ 7 & 1 & -3 \\ -1 & 5 & -3 \end{pmatrix}.$$

(d). (5 points) Find the solutions x_1, x_2 and x_3.

From $Ax = b$, we have $x = A^{-1}b$. Then

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1}\begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \frac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ 7 & 1 & -3 \\ -1 & 5 & -3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 10/3 \\ 17/6 \\ 13/6 \end{pmatrix}.$$

Problem 8. (20 points) By using the row or the column operators, find the determinant of the following matrix

$$A = \begin{pmatrix} 3 & -1 & 2 & 1 \\ -2 & 0 & 1 & -3 \\ -1 & 2 & -3 & 4 \\ -2 & 1 & -2 & 1 \end{pmatrix}.$$

(Note that, simply proving an answer by using calculators will receive little credit. Show your work here.)

$$|A| = \begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & -2 & 1 & -3 \\ -2 & -1 & -3 & 4 \\ -1 & -2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & 5 & 1 & 6 \\ 0 & 1 & 0 & 2 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 5 & 1 & 6 \\ 0 & -2 & 1 & -3 \end{vmatrix} = -\begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 5 \end{vmatrix} = -5.$$