Fall 2011, Math 290, Midterm III practice

Name (Print): (first)____________ (last)____________

Signature:

There are a total of 100 points on this 50 minutes exam. This contains 6 pages (including this cover page) and 8 problems. Check to see if any page is missing. Enter all requested information on the top of this page. Calculators may be needed. Please turn off cell phones. You are allowed to bring one-half of one single-sided 8.5×11 inch page of notes, in your own handwriting, to the exam. Do not give numerical approximations to quantities such as $\sin 5, \pi, e$ or $\sqrt{2}$. However you should simply $\sin \frac{\pi}{2} = 1$ and $e^0 = 1$, etc.

The following rules apply:

- To get full credit for a problem you must show the details of your work, in a reasonably neat and coherent way, in the space provided. Answers unsupported by an argument will get little credit. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically CORRECT and carefully and legibly written.
- NO books. No computers. Do all of your calculations on this test paper.

Problem	Score
1	
2	
3	
Total	

Write down the answers to the problems of multiple choice here.

Problem	1	2	3	4	5
Your answer					

Problem 1. (10 points) Let V be a vector space. Let u and v be two vectors in V. Determine which of the following statements is wrong.

(A) If $u + v = 0$, then 0 is a linear combination of u and v.

(B) If $u + v = 0$, then u is a linear combination of v.

(C) If $u + v = 0$, then u and v are linearly independent.

(D) If $u + v = 0$, then u and v are linearly dependent.

(E) None of the above.

Problem 2. (10 points) Determine which of the following statements is wrong.

(A) The set $\{(1, 0), (0, 1)\}$ is a basis for $\mathbb{R}^2$.

(B) If $\dim V = n$, then any set of $n - 1$ vectors in V must be linearly independent.

(C) The set $\{(1, 1), (1, 2)\}$ is a basis for $\mathbb{R}^2$.

(D) If $\dim V = n$, then there exists a set of $n + 1$ vectors in V that will span V.

(E) None of the above.

Problem 3. (10 points) Let A be an $m \times n$ matrix. Determine which of the following statements is wrong.

(A) The dimension of the row space of A is $\leq m$.

(B) The nullspace of A is a subspace of $\mathbb{R}^n$.

(C) If an $m \times n$ matrix A is row-equivalent to an $m \times n$ matrix B, then the row space of A is equivalent to the row space of B.

(D) The column space of a matrix A is equal to its row space.

(E) None of the above.

Problem 4. (10 points) Determine which of the following statements is right.

(A) Let $T : V \to W$ be a linear transform from a vector space V to another vector space W, then $T(-v) = -T(v)$.

(B) $T(x) = x + 1 : \mathbb{R} \to \mathbb{R}$ is a linear transform.

(C) The range of a linear transform T from a vector space V to another vector space W is not a subspace of W.

(D) A linear transform T from V to W is called onto if for any vector w in W, there exists a vector v in V such that $w = T(v)$.

(E) None of the above.

Problem 5. (10 points) Determine which of the following statements is wrong.

(A) Any linear transform T from $\mathbb{R}^n$ to $\mathbb{R}^m$ can be written as in the matrix form $Tv = Av$, where A is an $m \times n$ matrix and v is a vector in $\mathbb{R}^n$.

(B) The composition T of two linear transforms T_1 and T_2, defined by $T(v) = T_2(T_1(v))$, is defined if the range of T_1 lies within the domain of T_2.

(C) If T is a linear transform from $\mathbb{R}^n$ to $\mathbb{R}^n$ with the standard matrix A, then T is invertible if A is invertible.

(D) If T_1 and T_2 are both linear transforms from $\mathbb{R}^n$ to $\mathbb{R}^n$, then the compositions T_1T_2 and T_2T_1 have the same standard matrix.

(E) None of the above.

Problem 6 (10 points). Let $V = P_2$. Prove that the set $\{1, 2 + x, x^2 - x + 1\}$ is linearly independent.

Problem 7. Let $A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$.

Problem 7 (a). (8 points) Find a basis for the row space of A.

Problem 7 (b). (8 points) Find a basis for the nullspace $N(A) = \{x \in \mathbb{R}^3 : Ax = 0\}$.

Problem 7 (c). (4 points) Find the rank of A: $\operatorname{rank}(A)$, and the dimension of the nullspace of A: $\dim N(A)$.

Problem 8. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be given by

$$T(x_1, x_2, x_3, x_4) = (x_1 - 2x_2, x_2, x_3 + x_4, x_3).$$

Problem 8 (a). (10 points) Determine whether T is invertible. If so, determine $T^{-1}(x_1, x_2, x_3, x_4)$.

Problem 8 (b). (10 points) Find the expression for $T^{-2}(x_1, x_2, x_3, x_4)$.