FINAL EXAM REVIEW

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1. Introduction

This final exam consists of 10 problems. The first part is multiple choice problem, which contains 10 sub-problems. There are 9 writing problems from chapter 1,2,3,4,5, respectively.

A good way to review for the final is to look through the homework problems, midterms and the book. I will provide some of the key concepts in the following sections.

We know that elementary row operations have been appeared a lot. If the test problems ask you to solve a linear system by using the Gauss-Jordan elimination method, you need to show the detailed steps to reduce matrices to the row-echelon form, or the reduced row-echelon form. If the test problems ask you to find bases for the row spaces, the column spaces, or the null spaces of a matrix, you can use calculators to reduce the matrix to the row echelon form, or the reduced row echelon form.

2. Chapter 1

1. There are several important concepts in this chapter, for instance, the system of linear equations, elementary row operations, matrix, matrix multiplication, the inverse of a matrix.
2. There is a very important method, the Gauss-Jordan elimination. It is useful in solving the linear systems of equations by reducing the augmented matrix to the reduced row-echelon form or row echelon form. It is also useful in finding inverses of matrices by applying the elementary row operations on $[A|I]$ to $[I|A^{-1}]$.
3. We have introduced one way to find inverses of matrices, which is by use of the elementary row operations. Another way is to

use the adjoint matrix that involves the definition of minors and cofactors: if A is an invertible matrix of order n, or $\det(A) \neq 0$,

$$A\,\mathrm{adj}(A) = \det(A) I_n. \tag{1}$$

4. The concept of inverse matrices is only applied to square matrices.
5. About the algebraic operations on matrix, some fail on matrix multiplications. For instance, the product of two nonzero matrices might equal to zero; the cancellation law for matrix multiplication fails.

3. Chapter 2

This chapter is about determinants of matrices. Determinant of matrices is a number, and is only applied to square matrices. For any matrix of order 2, let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then

$$\det(A) = ad - bc.$$

1. One way to evaluate the determinant of a matrix is by use of cofactor expansions: we can expand the determinant along a row or a column.
2. Another way to evaluate determinants of a matrix is to apply elementary row operations to evaluate determinants.

- We used the method of elementary row operations to solve linear systems of equations. Another way is to use the Cramer's rule.

4. Chapter 3

1. The two dimensional or three dimensional Euclidean spaces are special examples of vector spaces. There are special structure in the case of Euclidean spaces, for example, we can define the inner product of two vectors, norm of vectors, and distance between two vectors.
2. An important concept is "orthogonality" defined by setting inner products equal to zero. There are several well-known rules associated with vectors in Euclidean spaces, for instance, the

parallelogram rule, the Cauchy-Schwarz inequality, the parallelogram equation for vectors, and the theorem of Pythagoras in Euclidean spaces $\mathbb{R}^n$.

5. Chapter 4

1. The definition of vector spaces is important. Review the examples in the book, or in the homework that are vector spaces, and are not vector spaces.
2. In this chapter, we have seen a lot of definitions, subspaces, linear dependence, linear independence, basis. We require that you are not only familiar with them but also know the method to prove them.
3. Suppose that we have two bases $\mathbf{B}$ and $\mathbf{B'}$, there are transition matrices associated with the change of bases, $P_{\mathbf{B}\to\mathbf{B'}}$ and $P_{\mathbf{B'}\to\mathbf{B}}$ such that
$$P_{\mathbf{B}\to\mathbf{B'}}P_{\mathbf{B'}\to\mathbf{B}} = I.$$

- There are definitions of row spaces, column spaces, and null spaces for matrices. With respect to those, there is the dimension theorem (or the rank-nullity theorem.)

6. Chapter 5

- We only have one section in this chapter. Given a square matrix A, $\lambda \in \mathbb{C}$ is called an eigenvalue of A if there exists a nonzero eigenvector $\mathbf{v}$ such that
$$A\mathbf{v} = \lambda\mathbf{v}, \text{ or}(A - \lambda I)\mathbf{v} = \mathbf{0}.$$
- For each eigenvalue λ, the solution space of $(A - \lambda I)\mathbf{x} = \mathbf{0}$ is the eigen-space. The way to find the basis for each eigen-space is to apply the Gauss-Jordan elimination method to find the basis for each null space of $(A - \lambda I)\mathbf{x} = \mathbf{0}$.

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