Math 290, Midterm key

Name (Print): (first)____________ (last)____________

Signature:

The following rules apply:

- There are a total of 20 points on this 50 minutes' exam. This contains 8 pages (including this cover page) and 9 problems. Check to see if any page is missing. Enter all requested information on the top of this page.
- To get full credit for a problem, you must show the details of your work in a reasonably neat and coherent way and in the space provided. Answers unsupported by an argument will get little credit.
- NO books. No computers. No cell phones. Do all of your calculations on this test paper.

Multiple Choice Problem	1	2	3	4	5	6	7	8
Answer	B	B	C	D	A	C	D	B

Problem 1. (2 points). Determine which of the following statements is true.

(A). $x^2 + 3y^2 = 4$ is a linear equation.

(B). The linear system $\begin{cases} x - y &= 1 \\ 2x + y &= 6 \end{cases}$ is consistent.

(C). The linear system $\begin{cases} 4x - 2y &= 1 \\ 16x - 8y &= 4 \\ x + y &= 2 \end{cases}$ is not consistent.

(D). The ordered pair $(\frac{8}{7}, 1)$ is the only solution to the linear equation $7x - 5y = 3$.

(E). None of the above.

Problem 2. (2 points). Determine which of the following statements is true.

(A). The following matrix is in the reduced row-echelon form: $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(B). The solutions to the linear equation $x - 5y + z = 4$ can be expressed parametrically as

$$x = 4 + 5s - t,\ y = s,\ z = t.$$

(C). The argumented matrix for the linear system $\begin{cases} x - 3y + 4z &= 7 \\ y + 2z &= 2 \\ x + z &= 5 \end{cases}$ is $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$.

(D). A homogeneous linear system with the same number of unknowns as the equations has infinitely many solutions.

(E). If a matrix is in the row echelon form, then it is also in the reduced row echelon form.

Problem 3. (2 points.) Determine which of the following statements is true.

(A). If A and B are matrices of 2×2, then

$$AB = BA.$$

(B). If A and B are square matrices of the same order, then

$$\operatorname{tr}(AB) = \operatorname{tr}(A)\operatorname{tr}(B).$$

(C). If A and B are square matrices of the same order, then

$$(AB)^T = B^T A^T.$$

(D). If A, B and C are square matrices of the same order such that $AC = BC$, then $A = B$.

(E). The diagonal entries of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ are 1 and 6.

Problem 4. (2 points.) Determine which of the following statements is true.

(A). If A and B are two matrices satisfying $AB = I$, then A is invertible, and the inverse is B.

(B). If $ad - bc \neq 0$, the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\frac{1}{ad - bc} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}.$$

(C). If A^2 is invertible, then A may not necessarily invertible.

(D). For all square matrices A and B of the same order, it is true that

$$(A + B)^2 = (A + B)(A + B) = A^2 + AB + BA + B^2.$$

(E). The sum of two invertible matrices of the same order must be invertible.

Problem 5. (2 points.) Determine which of the following statements is true.

(A). The matrix $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an elementary matrix.

(B). If A is an $n \times n$ matrix, then $Ax = 0$ only has one solution, $x = 0$.

(C). Any square matrix can be expressible as a product of elementary matrices.

(D). For a 3×3 matrix, after a sequence of elementary row operations, if the partitioned matrix $[A|I]$ reduces to the following form

$$\begin{bmatrix} 1 & 6 & 4 & | \; 1 & 0 & 0 \\ 0 & -8 & -9 & | -2 & 1 & 0 \\ 0 & 0 & 0 & | -1 & 1 & 1 \end{bmatrix}.$$

Then the matrix A is invertible.

(E). None of the above.

Problem 6. (2 points.) Determine which of the following statements is wrong.

(A). If A and B are $n \times n$ matrices such that $AB = I_n$, then $BA = I_n$.

(B). For any $b_1, b_2 \in \mathbb{R}$, the linear system $\begin{cases} x + 3y & = b_1, \\ -2x + y & = b_2 \end{cases}$ always has a solution.

(C). It is impossible for a system of linear equations to have exactly two solutions.

(D). If A is a square matrix and $b, c \in \mathbb{R}$, and if the linear system $Ax = b$ has a unique solution, then the linear system $Ax = c$ must also have a unique solution.

(E). None of the above.

Problem 7. (2 points.) Determine which of the following statements is true.

(A). The determinant of the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad + bc$.

(B). For any 3×3 matrix, we always have $C_{23} = C_{32}$ where C_{23} is the cofactor of the entry a_{23}, and C_{32} is the cofactor of the entry a_{32}.

(C). For all square matrices A and B, then it is true that

$$\det(A + B) = \det(A) + \det(B).$$

(D). For $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, then $\det(A) = 1 \times 1 \times 2 \times 4 = 8$.

(E). None of the above.

Problem 8. (2 points.) Determine which of the following statements is wrong.

(A). If A is a 4×4 matrix and B is obtained from A by interchanging the first two rows and then interchanging the last two rows, then

$$\det(B) = \det(A).$$

(B). If A is a 3×3 matrix, and B is obtained from A by adding 5 times the first row to each of the second and third rows, then

$$\det(B) = 25 \det(A).$$

(C). Let $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$, then

$$\det(A) = (b-a)(c-a)(c-b).$$

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(D). Let A be an $n \times n$ matrix, then A is invertible if and only if $\det(A) \neq 0$.

(E). Let A be an $n \times n$ matrix, and A is invertible, then

$$A^{-1} = \frac{1}{\det(\mathrm{A})} \operatorname{adj}(A),$$

where $\operatorname{adj}(A)$ is called the adjoint of A.

Problem 9. Use the method of the Gauss-Jordan elimination to solve the following linear system.

$$\begin{cases} 2x - y - 3z & = 1, \\ -x + 2y - 3z & = 2, \\ x + y + 4z & = 3. \end{cases}$$

(a). (1 point.) Write the linear system in the matrix form, $Ax = b$.

Proof.

$$\begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & -3 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

□

(b). (1 point.) Write the argumented matrix for $Ax = b$.

Proof.

$$\begin{bmatrix} 2 & -1 & -3 & 1 \\ -1 & 2 & -3 & 2 \\ 1 & 1 & 4 & 3 \end{bmatrix}$$

□

(c). (2 points.) Apply the method of Gauss-Jordan elimination to reduce the matrix in **(b)** to the reduced row echelon form and then obtain the solutions.

Proof. We apply the Gauss-Jordan elimination to reduce the augmented matrix to the reduced row echelon form.

$$\begin{bmatrix} 2 & -1 & -3 & 1 \\ -1 & 2 & -3 & 2 \\ 1 & 1 & 4 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & -3 & -11 & -5 \\ 0 & 3 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & -3 & -11 & -5 \\ 0 & 0 & -10 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & -3 & -11 & -5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & -3 & 0 & -5 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

This implies that

$$\begin{cases} x + y & = 3 \\ y & = \frac{5}{3} \\ z & = 0. \end{cases}$$

Solving this linear system, we obtain

$$x = \frac{4}{3}, \quad y = \frac{5}{3}, \quad z = 0.$$

□